# MORE EXERCISES FOR THE 2ND MIDTERM EXAM IN INDUSTRIAL ORGANIZATION

## (SOLUTIONS, MIGHT CONTAIN TYPOS OR MISTAKES)

**I. In a supply chain the producer and the retailer firms are both monopolists in the upstream (wholesale) and downstream (retail) markets, respectively.**

The demand curve is characterized by the equation P=5000-2Q, while the total cost of the upstream monopolist is given by the equation: TC(Q)=1000Q, and the total cost of the downstream mopopolist is given by the equation TC(Q)=r*Q, where r is the wholesale price of the product (as opposed to P, the retail price). The two firms are considering a merger that would result in an integrated firm that remains a monopolist.

> *Determine the pre-merger and the post-merger profits of the two firms! Would a merger be profitable for the owners of the firms?*

Downstream problem:
Downstream marginal cost: $MC_D = dTC_D/dQ = r = AVC_D$
$P = 5000 – 2Q$
$TR_D = PQ = 5000Q – 2Q^2$
$MR_D = dTR_D/dQ = 5000 – 4Q$
Optimum at MR=MC: $5000 – 4Q = r$

Upstream problem:
Upstream marginal cost: $MC_U = dTC_U/dQ = 1000 = AVC_U$
$r = 5000 – 4Q$
$TR_U = rQ = 5000Q – 4Q^2$
$MR_U = dTR_U/dQ = 5000 – 8Q$
Optimum at MR=MC: $5000 – 8Q = 1000$
$8Q = 4000 \Rightarrow Q = 500$
$r = 3000$
$P = 4000$
$\pi = (P – AVC)q – FC$
$\pi_U = (3000 – 1000)500 = 1$ M (M = million)
$\pi_D = (4000 – 3000)500 = 0.5$ M

Monopoly problem:
$MC = dTC/dQ = 1000$
$P = 5000 – 2Q$
$TR = PQ = 5000Q – 2Q^2$
$MR = dTR/dQ = 5000 – 4Q$
Optimum at MR=MC: $5000 – 4Q = 1000$
$Q = 1000$
$P = 3000$
$\pi_{integrated} = (3000 – 1000)1000 = 2\text{ M} > 1.5\text{ M} \Rightarrow$ Profitable to merge

**II. Two firms that compete according to the rules of Cournot competition would like to form a cartel to increase their profits.**

The demand curve is characterized by the equation $P=3200-2Q$, while the total cost for both firms are given by the equation: $TC(q)=800q$.

> *Is it possible to sustain this cartel, if they would like to collectively earn as much profit as one monopolist and use the simple trigger strategy to prevent deviations from the agreement, if the interest rate will be 25% and the probability of the collusion continuing for one more round is 60% in all rounds?*

Non-cooperative profit (punishment period):
$TR_1 = Pq_1 = 3200q_1 – 2Qq_1 = 3200q_1 – 2q_2q_1 – 2q_1q_1$
$MR_1 = dTR/dq_1 = 3200 – 2q_2 – 4q_1$
$MC_1 = dTC/dq_1 = 800 = AVC_1$
Optimum at MR=MC: $3200 – 2q_2 – 4q_1 = 800$
Simplifies to: $q_1 = 600 – q_2/2$

MCs are equal, thus quantities should also be equal in the equilibrium:
$q_1^* = 600 – q_1^*/2$
$q_1^* = 600/(1+1/2) = 400$
$Q^* = 2q_1^* = 800$
$P = 1600$
$\pi = (P – AVC)q – FC$
$\pi_N = (1600 – 800)400 = 0.32M$

Cartel profit (monopoly profits distributed evenly among firms):
$TR = PQ = 3200Q – 2Q^2$
$MR = dTR/dQ = 3200 – 4Q$
$MC = dTC/dQ = 800 = AVC$
Optimum at MR=MC: $3200 – 4Q = 800$
Simplifies to: $Q = 600 \Rightarrow P = 2000$; $q_1 = q_2 = 300$
$\pi_M = (2000 – 800)300 = 0.36M$

Cheating (unilateral deviation from the agreement):
$q_2 = 300$ (as agreed)
Conditional profit maximization: $q_1^* = 600 – q_2/2 = 450$
Thus: $Q = 300 + 450 = 750 \Rightarrow P = 1700$
$\pi_D = (1700 – 800)450 = 0.405M$

Stochastic discount factor evaluation:
$\Gamma = \rho/(1+r) = 0.6/1.25 = 0.48$

Cartel is unsustainable, since:

$$0.48 < \frac{\pi_D - \pi_M}{\pi_D - \pi_N} = \frac{9}{17} \approx 0.53$$

**III. Two firms that compete according to the rules of the Bertrand model with homogeneous products and no capacity constrains would like to form a cartel to increase their profits.**

The demand curve is characterized by the equation P=3200-2Q, while the total cost for both firms are given by the equation: TC(q)=800q.

> *Is it possible to sustain this cartel, if they would like to collectively earn as much profit as one monopolist and use the simple trigger strategy to prevent deviations from the agreement, if the interest rate will be 25% and the probability of the collusion continuing for one more round is 90% in all rounds?*

Non-cooperative profit (punishment period):
$p_1 = p_2 = MC = AVC = 800 \Rightarrow \pi_N = 0 = (800 - 800)*q$

Cartel profit (monopoly profits distributed evenly among firms):
$TR = PQ = 3200Q - 2Q^2$
$MR = dTR/dQ = 3200 - 4Q$
$MC = dTC/dQ = 800 = AVC$
Optimum at MR=MC: $3200 - 4Q = 800$
Simplifies to: $Q = 600 \Rightarrow P = p_1 = p_2 = 2000$; $q_1 = q_2 = 300$
$\pi_M = (2000 - 800)300 = 0.36M$

Cheating (unilateral deviation from the agreement):
$p_2 = 2000$ (as agreed)
Cheating: undercut the other firm by epsilon: $p_1^* = 2000 - \varepsilon$
Thus: $Q = q_1 = 600 + 0.5\varepsilon$
$\pi_D = (2000 - \varepsilon - 800)(600 + 0.5\varepsilon) = 0.72M - 0.5\varepsilon^2$

Stochastic discount factor evaluation:
$\Gamma = \rho/(1+r) = 0.9/1.25 = 0.72$

Cartel is sustainable, since:

$$0.72 > \frac{\pi_D - \pi_M}{\pi_D - \pi_N} = 0.5 + \mu(\varepsilon) \approx 0.5$$

**IV. Twenty-one out of twenty-nine firms in an industry with Cournot competition would like to merge into one with a view to increasing their profits.**

The demand curve is characterized by the equation P=12000-3Q, while the total cost for every firm in the industry is given by the equation: TC(q)=3000q+8000.

> *Would the merger increase the profits of the merging companies compared to the pre-merger state of affairs if we assume that their costs would be characterized by the same cost function and they would still compete with the non-merged firms according to the rules of the Cournot model?*

The pre-merger profits for all firms (N=29):

$$\pi^C = \frac{(A-c)^2}{(N+1)^2 B} - FC = \frac{(12000-3000)^2}{(29+1)^2 3} - 8000 = 22000$$

The post-merger profits for all firms (N=29; M=21):

$$\pi_m = \pi_{nm} = \frac{(A-c)^2}{(N-M+2)^2 B} - FC = \frac{(12000-3000)^2}{(29-21+2)^2 3} - 8000 = 262000$$

The merger would **not be profitable**, since 21· 22000 = 0.462M > 0.262M

**V. Three out of two firms in an industry with Cournot competition would like to merge into one with a view to increasing their profits.**

The demand curve is characterized by the equation P=16000-2Q, while the total cost for every firm in the industry is given by the equation: TC(q)=8000q.

> *Would the merger increase the profits of the merging companies compared to the pre-merger state of affairs if we assume that their costs would be characterized by the same cost function and they would become a Stackelberg leader with the non-merged firm behaving as a Stackelberg follower?*

The pre-merger profits for all firms (N=3):

$$\pi^C = \frac{(A-c)^2}{(N+1)^2 B} - FC = \frac{(16000-8000)^2}{(3+1)^2 2} = 2M$$

The post-merger profits for a Stackelberg leader in a duopoly:

$$\pi_m = \frac{(A-c)^2}{8B} - FC = \frac{(16000-8000)^2}{16} = 4M$$

The merger would **be indifferent (neutral)**, since 2· 2M = 4M

**VI. Two firms in a Stackelberg duopoly would like to merge into one with a view to increasing their profits.**

The demand curve is characterized by the equation $P=20000-2Q$, while the total cost for the leader firm in the industry is given by the equation: $TC(q_L)=8000q_L$, that of the follower firm is given by $TC(q_F)=10000q_F$.

> *Would the merger increase the collective profits of the merging companies compared to the pre-merger state of affairs if we assume that their costs would be characterized by the cost function of the more efficient firm and they could keep their monopoly indefinitely?*

Market demand $P = 20000 – 2Q$ can be rewritten as $P = 20000 – 2q_L – 2q_F$

<u>The follower's problem</u>:
$TR_F = Pq_F = 20000q_F – 2q_Lq_F – 2q_Fq_F$
$MR_F = dTR/dq_F = 20000 – 2q_L – 4q_F$
$MC_F = dTC/dq_F = 10000 = AVC_F$
Optimum at MR=MC: $20000 – 2q_L – 4q_F = 10000$
Simplifies to: $q_F = 2500 – q_L/2$

<u>The leader's problem</u>:
$TR_L = Pq_L = 20000q_L – 2(2500 – q_L/2)q_L – 2q_Lq_L$
Simplifies to: $TR_L = 15000q_L – q_Lq_L$
$MR_L = dTR/dq_L = 15000 – 2q_L$
$MC_L = dTC/dq_L = 8000 = AVC_L$
Optimum at MR=MC: $15000 – 2q_L = 8000$
Simplifies to: $q_L = 3500$ ➔ $q_F = 750$ ➔ $Q = 4250$ ➔ $P = 20000 – 8500 = 11500$
$\pi_L = (11500 – 8000)3500 = 12.25$ M
$\pi_F = (11500 – 10000)750 = 1.125$ M

<u>Monopoly problem</u>:
$MC = dTC/dQ = 8000$
$P = 20000 – 2Q$
$TR = PQ = 20000Q – 2Q^2$
$MR = dTR/dQ = 20000 – 4Q$
Optimum at MR=MC: $20000 – 4Q = 8000$
$Q = 3000$
$P = 14000$

$\pi_{integrated} = (14000 – 8000) \cdot 3000 = 18$ M > 12.25 M + 1.125 M ➔ Profitable to merge